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# A methodology for assessing longevity basis risk

## User Guide

This user guide relates to research prepared by Cass Business School and Hymans Robertson LLP for the Institute and Faculty of Actuaries (IFoA) and the Life and Longevity Markets Association (LLMA). The IFoA and the LLMA are joint funders of this project.


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# Reliances & Limitations

This User Guide has been produced by Hymans Robertson LLP and Cass Business School for the Longevity Basis Risk Working Group (LBRWG) of the Institute & Faculty of Actuaries (IFoA) and the Life & Longevity Markets Association (LLMA). This User Guide is designed to help practitioners gain a broad understanding of the methodology we are proposing for assessing (demographic) basis risk. Further information and details of the limitations and practicalities of the methodology are provided in our report 'A methodology for assessing longevity basis risk' presented to the Institute & Faculty of Actuaries on 8 December 2014.

When reading both this User Guide and our main report, please be aware that the scope of this phase of work is limited to producing a proposed methodology for assessing (demographic) basis risk. For example identification and development of appropriate metrics for assessing basis risk, quantification of potential capital savings and presentation of basis risk results to regulatory authorities are excluded from this initial phase and (potentially) form part of a secondary phase of this project.

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## Research sponsored by IFoA and LLMA

The research on which this user guide is based was funded by the Institute & Faculty of Actuaries (IFoA) and the Life & Longevity Markets Association (LLMA). The team from Cass Business School and Hymans Robertson LLP thank the IFoA and the LLMA for their support in this work.

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# Foreword

On behalf of the joint Longevity Basis Risk Working Group (LBRWG) established by the Life and Longevity Markets Association (LLMA) and the Institute and Faculty of Actuaries (IFoA), I am delighted to introduce this user guide.

This guide provides a high level summary of the methodology developed on behalf of the LBRWG to assess longevity basis risk. It is designed to complement the more detailed technical paper available on the IFoA website. Together these documents form the key outputs of the first phase of a longevity basis risk project commissioned and funded by the IFoA and the LLMA, and undertaken on our behalf by Cass Business School and Hymans Robertson LLP.

## The importance of longevity basis risk

Longevity basis risk arises because different populations, or subpopulations, will inevitably experience different longevity outcomes. This is a significant issue for those wishing to hedge longevity risk using a published mortality index – whether they be pension schemes, insurers, reinsurers or banks. To put it simply, actual longevity outcomes, and therefore cashflows, of the hedged portfolio will differ from those under the hedging instrument.

In addition, longevity basis risk can also present a wider issue for insurers using, in their reserving models, external data, such as population data, rather than their own policy data. The need to quantify and reserve for any potential basis risk is receiving increasing focus, particularly under Solvency II.

## Demographic aspects of longevity basis risk

There are several aspects of longevity basis risk. This research focuses on the impact of demographic and socio-economic differences between the portfolio and the index population, which can lead to different initial rates and trends in mortality. To date, there has been no well-established methodology for assessing these demographic aspects of longevity basis risk.

## Historical differences demonstrate the need to assess basis risk

A review of existing literature and analysis of pension scheme data have provided evidence that historic mortality improvement rates have varied by socio-economic class and deprivation. These variations have been significant and sometimes as large as the variation by gender. This demonstrates the significance of demographic basis risk and confirms the need to model longevity basis risk.

## The need for a two-population model

To be able to assess demographic basis risk, the required model needs to able to capture the mortality trends in both the reference population backing the hedging instrument and in the population of the portfolio being hedged. Given this model, the assessment of other aspects of basis risk, such as sampling risk and structuring risk, becomes (in theory, at least) more straightforward.

## Delivering a framework to assess longevity basis risk

I am delighted that the research has delivered a framework for assessing longevity basis risk. This recognises the fact that different users, with different portfolios, will have different constraints on the models they can use in practice. The research has identified specific models and techniques for different situations, which we believe will provide a good starting point for assessing basis risk.

We are delighted to be able to present this research and hope it will prove of value to practitioners and enable an important step change in the ability to assess longevity basis risk.

**Pretty Sagoo**
Chair of the LLMA and IFoA Joint Longevity Basis Risk Working Group

# Contents



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# 1 Introducing our methodology

Early in our research we recognised that no single model would be capable of meeting the needs of all users. Our research has therefore focused on identifying a small number of models which should provide a good starting point to most users, together with a set of rules to enable users to identify which model(s) to focus on.

These rules have been collated into the decision tree provided overleaf. Section 2 of this guide provides more detailed explanation on the questions contained within the decision tree.

## Directly modelling basis risk

A key question in the decision tree relates to whether a book is 'self-credible' (i.e. has a large number of lives and sufficient back history). If it is, then it is possible to parameterise a two-population model directly from mortality experience data relating to i) the population underlying the index and ii) the book population.

Our systematic assessment of candidate two-population mortality models identified two particular 'best of breed' models (specifically the M7-M5 model, or in some situations the CAE+Cohorts model). In section 3 we describe the M7-M5 model in more detail.

## Indirectly modelling basis risk – Characterisation approach

If the book is not 'self-credible', (i.e. it does not have a sufficiently large number of lives or lacks a sufficient back history) then it is not possible to robustly parameterise the book element of a two-population model directly from mortality experience data. In this situation an alternative approach is required.

The alternative we propose, which we describe as a "characterisation approach" enables an assessment of basis risk based on the characteristics of the book in question; leveraging an alternative larger dataset to provide the required volumes and back history of data.

This approach is described in section 4.

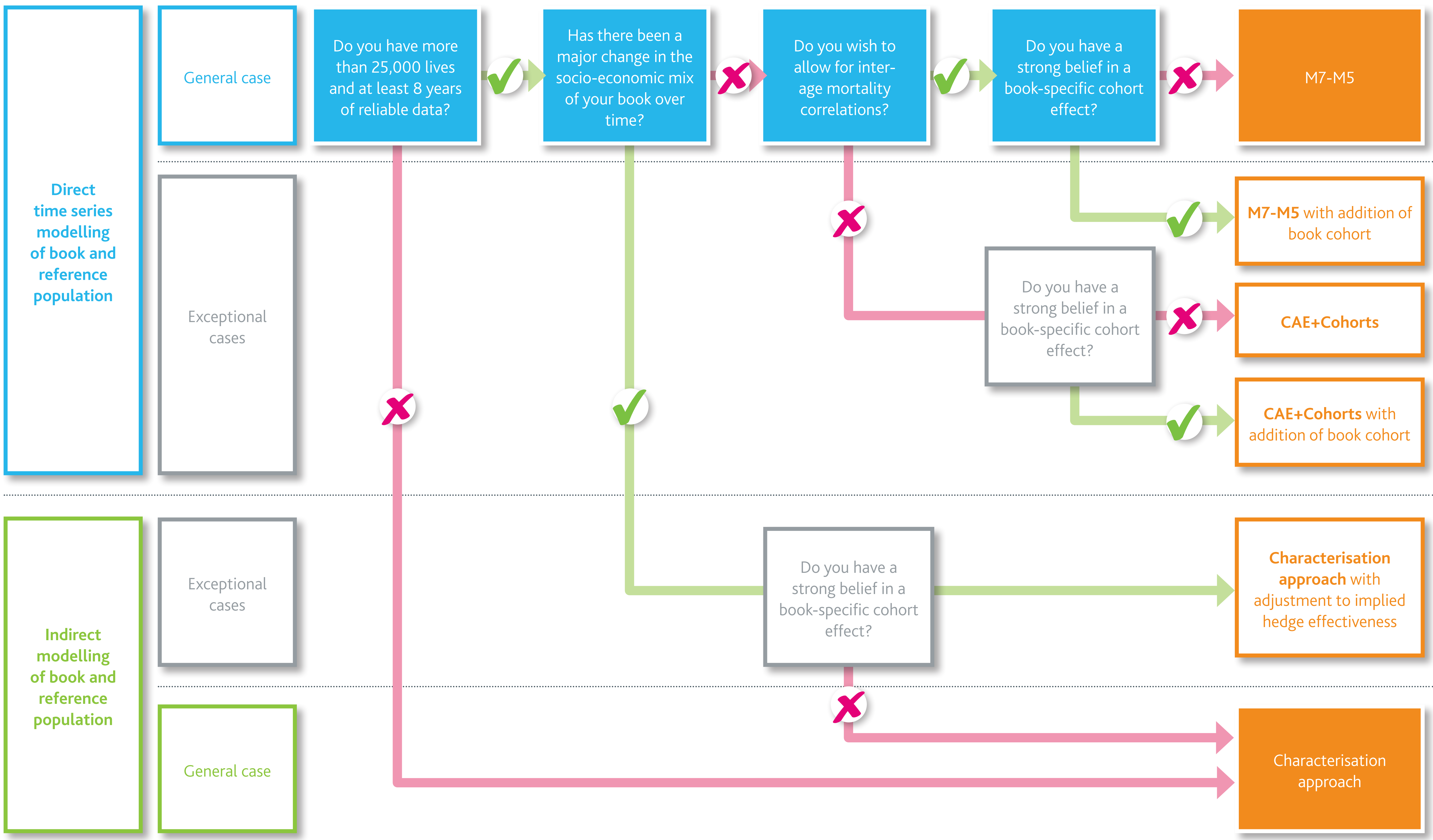
## Practicalities

Modelling longevity basis risk can be complex! We have sought to keep the models presented as simple as is reasonable.

Section 5 therefore moves on to explore a number of practical considerations for when the models are applied in the real world – and importantly how those practical issues can be addressed.

Please note that this User Guide is designed to provide an overview of our proposed framework for assessing Longevity Basis Risk – more detailed information on each of the models described here, and the analysis supporting our decisions can be found in our accompanying more detailed technical report 'Longevity Basis Risk - A methodology for assessing basis risk' presented to the Institute & Faculty of Actuaries on 8 December 2014.

# Choosing a method for modelling demographic basis risk



# 2 Answering the questions in the decision tree

## Do you have more than 25,000 lives and at least 8 years of reliable data?

This question is important as it determines whether there is sufficient historical data in your book to fit models directly to its experience, or whether an alternative 'indirect' approach is required.

Having a large number of lives is important to provide confidence in the relative shape of mortality of your book and the reference population. The exact number of lives is not a hard cut-off, but you will need 20-25,000 lives in order to prevent uncertainty in the parameters of the model falsely swamping the assessment of basis risk. It is important to also realise that if, for example, you are entering an index-based swap for men and women separately that this requirement applies to the data you have for each gender, rather than at the aggregate level.

A reasonable length history of book experience is needed in order to reliably fit a time series to the historical data, and then to project future possible trends in the book relative to the reference population. Empirical testing has shown that a minimum of 8 years is needed – below this length the modelling results are unreliable.

## Has there been a major change in the socio-economic mix of your book over time?

Generally we expect users to answer **no** to this question.

This question is important because where a book has seen a dramatic shift in its socio-economic mix, its historical mortality experience will not just be capturing basis risk, but also the shifting socio-economic mix. Using the book data to project the future mortality of the book relative to the reference population would extrapolate a continued shift in the socio-economic mix in the future. Typically we expect users to be interested in measuring longevity risk in relation to a specific book of lives, so projecting a continued change would be inappropriate.

Answering this question is likely to be a matter of judgement – but insurance companies who have materially changed their target market for annuity sales in recent years, and pension schemes where the sponsoring employer has fundamentally changed its core business might answer this question **yes**. For those books a characterisation approach based upon the current mix of lives is likely to be a more relevant starting point.



## Do you wish to allow for inter-age mortality correlations?

Generally we expect users to answer **yes** to this question.

For example, when using the methodology to structure / assess an index-based longevity swap it is important to assess how well the age structure of the swap provides protection for the (potentially different) age structure of the book. To do so it is necessary to capture the correlations that are likely to exist between mortality rates at different ages to avoid counter-intuitive conclusions.

In some circumstances though it might be appropriate to answer **no** to this question and use the alternative model (called CAE+Cohorts) suggested by the decision tree. For example, this would include circumstances where the user is keen to get an indicative assessment of the reduction in basis risk without necessarily considering in detail the precise structuring of the instruments held.

## Do you have a strong belief in a book-specific cohort effect?

Generally we expect users to answer **no** to this question.

The consequence of answering no is that the cohort effect modelled within your book will be the same as that of the reference population (and driven by the reference population data). This means that modelled basis risk will principally arise from differences in year on year improvements (and how these impact different ages), rather than effects specific to certain birth generations. We believe this is unlikely to be a concern since our work on fitting models for a wide range of different books has shown that including a cohort effect which is specific to the book has little impact on the quality of fit of the model, but materially increases the number of parameters to fit.

However, there may be cases where you do have a strong belief that the cohort effect will be materially different within your book to a reference population, such as the general England & Wales population. An example might be where the book relates to the pension scheme of a cigarette manufacturer, where smoking cessation patterns might have been very different to the national population. In such cases we would suggest that any book-specific cohort effect should have a parametric form as it is unlikely that users' data would support the fitting of a non-parametric form without considerable parameter uncertainty.

# 3 The M7-M5 model

We anticipate that most users whose books are of sufficient size to be 'self-credible' will follow the top line of the decision tree leading them to the M7-M5 model, a two population extension of the well-known CBD model of mortality.

This model has four parts:

1. A formula for the mortality rates in the reference population (the 'M7' bit)
2. A formula for the **difference** in mortality rates between the book population and the reference population (the 'M5' bit)
3. Some criteria to ensure that there is a unique fit of the model to your data, known as 'identifiability constraints'
4. A time series projection

## Reference population

$\text{logit}\ q = \log\left(\frac{q}{1-q}\right)$

This converts the mortality rates onto a scale in which they are broadly linear.

$\left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,R)}$

A quadratic term providing additional shape by age where $\sigma_x^2$ is the average value of $(x-\bar{x})^2$

$\gamma^R_{t-x}$

This allows for the cohort effect.

$$\text{logit}\ q^R_{xt} = \kappa_t^{(1,R)} + (x-\bar{x})\,\kappa_t^{(2,R)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,R)} + \gamma^R_{t-x}$$

$q^R_{xt}$

One year probability of death at age $x$ at time $t$.

$\kappa_t^{(1,R)} + (x-\bar{x})\,\kappa_t^{(2,R)}$

This provides a linear form to mortality with the $\kappa_t^{(1,R)}$ representing the level of mortality at time $t$ at the average age $\bar{x}$, and the $\kappa_t^{(2,R)}$ the slope of mortality with age.

## Book population

$$\text{logit}\ q^B_{xt} - \text{logit}\ q^R_{xt} = \kappa_t^{(1,B)} + (x-\bar{x})\,\kappa_t^{(2,B)}$$

$\text{logit}\ q^B_{xt} - \text{logit}\ q^R_{xt}$

This is the difference in one year probabilities of death between the book (B) and the reference population (R) on the logit scale.

$\kappa_t^{(1,B)} + (x-\bar{x})\,\kappa_t^{(2,B)}$

The difference in mortality is linear with age $x$ for each calendar year $t$. $\kappa_t^{(1,B)}$ represents the level of difference in mortality at time $t$ at the average age $\bar{x}$, and the $\kappa_t^{(2,B)}$ the slope of mortality difference with age.



## Identifiability constraints

In order to ensure that we can fit the two formulae to historical mortality data for the book and the reference population with a unique solution, we need to impose some constraints on the parameters. Specifically:

$$\sum_c \gamma^R_c = 0 \qquad \sum_c c\,\gamma^R_c = 0 \qquad \sum_c c^2\gamma^R_c = 0$$

:where c = t - x represents the birth years available in the data.

## Time series projection

When projecting future mortality an assumption needs to be made for how the five $\kappa_t$ terms in the above formulae evolve into the future. The assumption often made for the reference population is that $\kappa_t^{(1,R)}$, $\kappa_t^{(2,R)}$ and $\kappa_t^{(3,R)}$ follow a multi-variate random walk with drift i.e.:

$$\begin{bmatrix}\kappa_t^{(1,R)}\\ \kappa_t^{(2,R)}\\ \kappa_t^{(3,R)}\end{bmatrix} = \begin{bmatrix}d_1\\ d_2\\ d_3\end{bmatrix} + \begin{bmatrix}\kappa_{t-1}^{(1,R)}\\ \kappa_{t-1}^{(2,R)}\\ \kappa_{t-1}^{(3,R)}\end{bmatrix} + \xi^R_t$$

$d_1, d_2, d_3$

The year on year 'drift' or trend in each of the $\kappa_t^{(i,R)}$ terms; fitted to the historical data.

$\xi^R_t \sim N(0, \Sigma^R)$

This provides white noise, with the $\Sigma^R$ term a 3x3 variance-covariance matrix which enables the three time components to move in a correlated way. This is fitted to the historical data.

For the book population a commonly used assumption is that the time indices $\kappa_t^{(1,B)}$ and $\kappa_t^{(2,B)}$ follow a Vector Autoregressive Process i.e.:

$$\begin{bmatrix}\kappa_t^{(1,B)}\\ \kappa_t^{(2,B)}\end{bmatrix} = \begin{bmatrix}\phi_{01}\\ \phi_{02}\end{bmatrix} + \phi_1\begin{bmatrix}\kappa_{t-1}^{(1,B)}\\ \kappa_{t-1}^{(2,B)}\end{bmatrix} + \xi^B_t$$

$\phi_{01}, \phi_{02}$

Along with $\phi_1$ these determine the long term difference between book and reference mortality

$\phi_1$

A **matrix** which determines the speed of reversion to the long term mean difference

$\xi^B_t \sim N(0, \Sigma^B)$

This provides 'white noise', with the $\Sigma^B$ term a 2x2 variance-covariance matrix.

We have adopted these approaches.

However, our research has highlighted that it is important for users to engage with the choice of time series for the book population. In particular, care is needed to:

1. Verify that the historical data on differences between your book population and the reference population is consistent with your choice of time series.
2. Understand the assumptions implicit in the choice of time series – for example the time series shown here implies that, in the long-run:
   2.1. The central trend in future mortality improvements in the reference population is extrapolated from past experience, although with increasing uncertainty;
   2.2. The spread between the logit of mortality for the book and the reference population will revert from the current level to the average level seen historically; and
   2.3. The variance of difference in (logit) mortality between the book and the reference population is bounded, limiting the width of the 'funnels of doubt' for the difference in mortality between the book and the reference populations.

> **We anticipate that most users whose books are** of sufficient size to be 'self-credible' will follow the top line of the decision tree leading them to the M7-M5 model.



# 4 The characterisation approach

If the book is not 'self-credible', (i.e. it does not have a sufficiently large number of lives or lacks a sufficient back history) then it is not possible to robustly parameterise the book element of a two-population model directly from mortality experience data. In this situation an alternative approach is required.

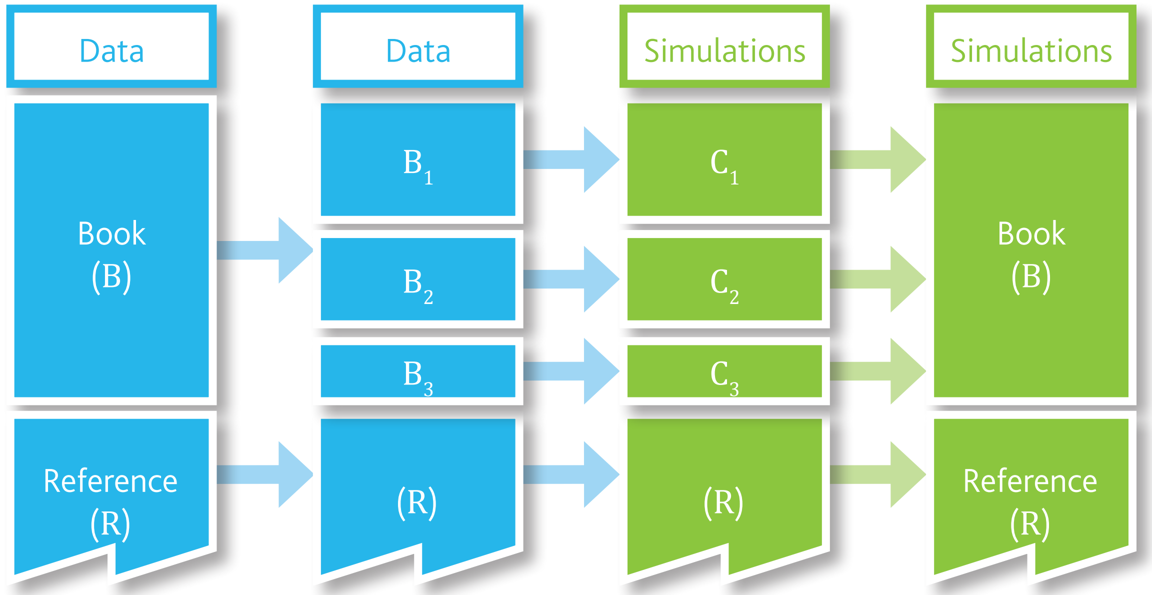
Indeed, even where the book is sufficiently large and with long enough experience history to use direct modelling, an alternative indirect approach may still be useful; either as a pragmatic initial assessment of the quantum of basis risk, or as an alternative approach as part of considering model risk.

The indirect alternative we propose, which we describe as a "characterisation approach" enables an assessment of basis risk based on the characteristics of the book in question; leveraging an alternative larger dataset to provide the required volumes and back history of data.

Instead of using the experience data of the book itself, the basic principle of the characterisation approach is to map the book onto a small number of characterising groups which:

- capture key aspects of demographic risk; and
- can be projected using an alternative data source with a reliable and longer back-history of mortality experience.

Schematically, this approach can be thought of in terms of the picture below. In this example the book population B is subdivided into three distinct sub-groups $B_1$, $B_2$ and $B_3$, according to some characterising criteria. Both B and the sub-populations $B_1$, $B_2$ and $B_3$ are too small for direct modelling. However, a larger characterising population C is available, and has previously been segmented (using the same characterising criteria) into sub-groups $C_1$, $C_2$ and $C_3$. Importantly, C has been chosen such that $C_1$, $C_2$ and $C_3$ are sufficiently large for direct modelling (in conjunction with the reference population R).




It is now possible to simulate survivorship in B indirectly, by first simulating future mortality rates for $C_1$, $C_2$ and $C_3$ and then mapping those simulations across to $B_1$, $B_2$ and $B_3$.

To use this approach we need to:

1. Identify a suitable characterising population (C)
2. Subdivide this population into appropriate groups (here three, $C_1$, $C_2$ and $C_3$, but can be any number)
3. Build a model to simulate these groups simultaneously along with the reference population

## Choosing the characterising population (C)

The dataset used for the characterising population needs to be large and have sufficiently long back history in order for it to give reliable simulations. Specifically, it will ideally be an order of magnitude greater than the minimum threshold for direct modelling, i.e. of the order of 250,000 lives, so that it can support direct modelling on the characterising sub-populations.

In order to be useful for the characterisation approach, the dataset needs to have sufficient information to allow segmentation into sub-groups that are likely to capture potential future longevity variations. In addition, the variables used for segmentation must be available and have a consistent definition / meaning with their use in your book population.

Potential datasets include:

- ONS data (split for example by a socio-economic variable such as postcode based index of multiple deprivation);
- CMI dataset (e.g. the SAPS data which has a back history of experience data split by pension amount); and
- Third party licensed datasets which can be split by a range of affluence and postcode metrics (e.g. Club Vita's dataset of DB pensioners).

## Choosing the characterising groups ($C_1$, $C_2$, etc...)

Having chosen our characterising population we need to identify how to segment it into groups which we believe will capture most of the heterogeneity in future longevity trends and thus demographic risk. A natural starting point in this regard would be differences in historical improvements, although the user may also wish to keep certain groups separate where he or she has a particular belief regarding the potential for divergent trends between those groups.

In our report we suggest six core principles that should be applied and balanced when choosing the characterising groups – along with statistical methods for applying these principles:

| | | |
|---|---|---|
| Credible size groups | Separate clear differences in improvements | Separate clear differences in mortality levels |
| Manageable number of groups | Group where similar improvements | Groups can be 'interpreted' |

## Modelling the groups and reference population

In isolation, each of the characterising groups can be thought of as a large book with a long back history and stable socio-economic mix. The decision tree would suggest applying the M7-M5 model to simulate each of the characterising groups.

It is likely though that there will be some correlation in the improvements seen for each of the characterising groups. To allow for this we propose using an extension of the M7-M5 model whereby an allowance is made for correlation. Specifically:

$$\text{logit } q^R_{xt} = \kappa_t^{(1,R)} + (x-\bar{x})\kappa_t^{(2,R)} + ((x-\bar{x})^2 - \sigma_x^2)\,\kappa_t^{(3,R)} + \gamma^R_{(t-x)}$$

$$\text{logit } q^{C_i}_{xt} - \text{logit } q^R_{xt} = \kappa_t^{(1,C_i)} + (x-\bar{x})\kappa_t^{(2,C_i)}$$

This is the same basic form as the M7-M5 model introduced in section 3. However, we now have one M5 equation per charactering sub-population so need to specify a multivariate time series with more terms, namely:

$$\kappa_t^C = (\kappa_t^{(1,C_1)}, \kappa_t^{(1,C_2)}, \ldots, \kappa_t^{(1,C_n)}, \kappa_t^{(2,C_1)}, \kappa_t^{(2,C_2)}, \ldots, \kappa_t^{(2,C_n)})^T$$

where $n$ is the number of characterising groups i.e. instead of two terms in the M7-M5 book population time series to model, we now have $2n$. This leads to a more complex fitting process – but has the advantage that it is a 'one-off' exercise and once it has been completed it can be applied to multiple books.



# 5 Practicalities

When using our methodology we anticipate there are a number of additional practical considerations users will want to make.

## Men and women

The models described here implicitly assume that the reference population and the book population can be treated as a single group of lives. This might be appropriate, for example when the reference population underlying the index is the entire England & Wales population. In practice most indices are split between men and women, and users are likely to want to hedge a book containing both men and women.

The exact approach depends on the type of hedge considered, but it will generally require modelling the reference population for men and women concurrently. This introduces an additional set of correlations (akin to the way we introduce correlations between the characterising groups). (See sections 9.2.2 and 12.2 of our main report for various approaches to address this.)

## The past as a guide to the future

Each of the models within the decision tree use time series to project into the future. These time series are usually calibrated to historic data – but in doing so they implicitly assume that past differences between the book (or the characterising groups) and the reference population are a guide to what we can expect to see in the future.

The incorporation of **user judgement** is important at this stage – key questions to ask yourself include:

1. Should the variability between the book and the reference population be bounded?
2. Do you wish to extrapolate any linear trends in the historic differences into the future? Or have the relative mortality rates tend to a stable level?

Answers to these questions will strongly influence the choice of time series to use. More information is provided in sections 9.1.1, 9.1.2 and 9.2.3 of our main report.

Please note that definitive guidance on the choice of time series is an area for future development.

## Own base table

We appreciate that generally users will have an existing view of the base mortality for their book. This can be embedded into the methodology.

For example, under the M7-M5 model you can calculate the implied annual improvements for the book and then apply these to your existing view of base mortality. (See section 9.2.1.2 of our main report.)

## Characterisation approach

When using the characterisation approach it is important to consider carefully the choice of data and characterising groups used. Specifically:

- **Relevance**: Is the dataset relevant to the lives in your book? Do the characterising variables (e.g. pension amount) have a consistent meaning in your book?
- **Availability**: Can you reliably map the majority of your membership onto the characterising groups?

We explore these further in section 12 of our main report.

## Other considerations

Other practical considerations such as the treatment of older ages, alternative reference populations to that of England & Wales (the focus of our research) and the forecasting horizon are explored in section 9 of our main report.

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